Note Title 5/7/2019

$$X(z) = \frac{z+4}{z^2+3z+2}$$

M
N

$M < N$

You can choose not to divide by z

Solution (1)

$$X(z) = \frac{z+4}{(z+2)(z+1)} = \frac{A}{z+2} + \frac{B}{z+1}$$

$$A = (z+2)\,X(z)\Big|_{z=-2} = (z+2)\left(\frac{z+4}{(z+2)(z+1)}\right)\Big|_{z=-2}$$

$$= \frac{2}{-1} = -2$$

$$B = (z+1)\,X(z)\Big|_{z=-1} = (z+1)\left(\frac{z+4}{(z+2)(z+1)}\right)\Big|_{z=-1}$$

$$= \frac{3}{1} = 3$$

$$\therefore X(z) = \frac{-2}{z+2} + \frac{3}{z+1}$$

$$= -2z^{-1}\left(\frac{z}{z+2}\right) + 3z^{-1}\left(\frac{z}{z+1}\right)$$

$$\therefore x[n] = -2(-2)^{n-1}u[n-1] + 3(-1)^{n-1}u[n-1]$$

Solution 2

$$\frac{X(z)}{z} = \frac{z+4}{z(z+3z+2)} = \frac{A}{z} + \frac{B}{z+2} + \frac{C}{z+1}$$

$$A = z\left(\frac{X(z)}{z}\right)\Big|_{z=0} = \frac{z+4}{(z+2)(z+1)}\Big|_{z=0} = \frac{4}{(2)(1)} = 2$$

$$B = (z+2)\left(\frac{X(z)}{z}\right)\Big|_{z=-2} = \frac{z+4}{z(z+1)}\Big|_{z=-2} = \frac{2}{(-2)(-1)} = 1$$

$$C = (z+1)\left(\frac{X(z)}{z}\right)\Big|_{z=-1} = \frac{z+4}{z(z+2)}\Big|_{z=-1} = \frac{3}{(-1)(1)} = -3$$

$$\frac{X(z)}{z} = \frac{2}{z} + \frac{1}{z+2} - \frac{3}{z+1}$$

$$X(z) = 2 + \frac{z}{z+2} - \frac{3z}{z+1}$$

$$x[n] = 2\delta[n] + (-2)^n u[n] - 3(-1)^n u[n]$$

	Solution ① $-2(-2)^{n-1}u[n-1] + 3(-1)^{n-1}u[n-1]$	Solution 2 $2\delta[n] + (-2)^n u[n] - 3(-1)^n u[n]$
$n=0$	0	0
$n=1$	1	1
$n=2$	1	1
$n=3$	-5	-5

Ex

$$X(z) = \frac{z^2 + 7z + 10}{z^2 + 7z + 12}$$

$M = N$

we need to divide by z

$$\frac{X(z)}{z} = \frac{(z+2)(z+5)}{z(z+4)(z+3)}$$

$$\frac{X(z)}{z} = \frac{A}{z} + \frac{B}{z+4} + \frac{C}{z+3}$$

$$X(z) = \frac{z^3 + 7z^2 + 10}{z^2 + 7z + 12} \qquad M > N$$

$$z^2 + 7z + 12 \;\overline{)\; z^3 + 7z^2 + 10}$$

$$z^3 + 7z^2 + 12z$$

$$10 - 12z$$

$$X(z) = z + \frac{10 - 12z}{z^2 + 7z + 12}$$

$$\delta[n] \leftrightarrow 1$$

$$\delta[n-q] \leftrightarrow z^{-q}$$

$$\delta[n-2] \leftrightarrow z^{-2}$$

$$\delta[n+2] \leftrightarrow z^{2}$$

$$\delta[n+1] \leftrightarrow z$$

$$x[n] = n\, e^{-bn} \cos(\Omega n)\, u[n]$$

$$\cos^2(\Omega n)\, u[n] = \cos(\Omega n)\cos(\Omega n)\, u[n]$$

$$\cos(\Omega n)\, u[n] \leftrightarrow \frac{z^2 - \cos(\Omega) z}{z^2 - 2\cos(\Omega) z + 1}$$

$$\cos^2(\Omega n) u[n] \leftrightarrow \frac{1}{2}\left[\frac{(z e^{j\Omega})^2 - \cos(\Omega)(z e^{j\Omega})}{(z e^{j\Omega})^2 - 2\cos(\Omega) z e^{j\Omega} + 1} + \right.$$

$$\frac{(ze^{-j\Omega})^2 - \cos(\Omega)(ze^{-j\Omega})}{(ze^{-j\Omega})^2 - 2\cos(\Omega)ze^{-j\Omega} + 1}$$

$$X(z) = \frac{z+4}{z^2+2z+3}$$

find the initial and final values

$$= \frac{\frac{1}{z} + \frac{4}{z^2}}{1 + \frac{2}{z} + \frac{3}{z^2}}$$

$$x[0] = \lim_{z\to\infty} X(z) = \lim_{z\to\infty}\left(\frac{z+4}{z^2+2z+3}\right) = 0$$

$$\lim_{n\to\infty} x[n] = \lim_{z\to 1}(z-1)X(z) = \lim_{z\to 1}(z-1)\left(\frac{z+4}{z^2+2z+3}\right) = 0$$

$$y[n] + y[n-1] - 2y[n-2] = 2x[n] - x[n-1]$$

Find $H(z)$

$$H(z) = \frac{z-z^{-1}}{1+z^{-1}-2z^{-2}} = \frac{z^2-z}{z^2+z-2}$$

$$Y(z) + z^{-1}Y(z) - 2z^{-2}Y(z) = 2X(z) - z^{-1}X(z) \qquad * z^2$$

$$z^2Y(z) + zY(z) - 2Y(z) = 2z^2X(z) - zX(z)$$

$$Y(z)[z^2+z-2] = X(z)[2z^2-z]$$

$$H(z) = \frac{2z^2-z}{z^2+z-2}$$

$$H(j\Omega) \quad / \quad Y(j\Omega) + Y(j\Omega)e^{-j\Omega} - 2Y(j\Omega)e^{-j2\Omega} = 2X(j\Omega) - X(j\Omega)e^{-j\Omega}$$

$$2\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 5y(t) = 6x(t) - 2\frac{dx(t)}{dt}$$

① Find $H(s)$

② Find $H(j\omega)$

①

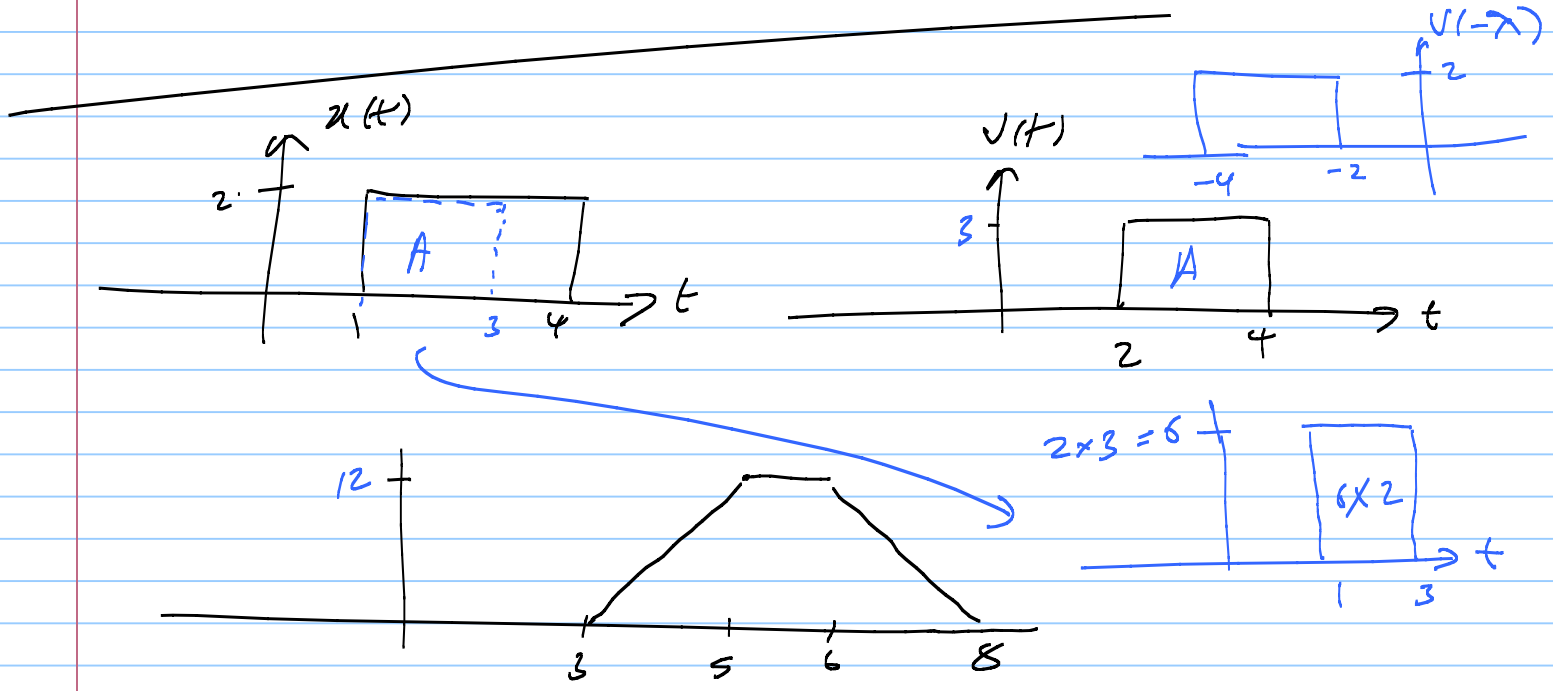
$$2s^2Y(s) + 3sY(s) + 5Y(s) = 6X(s) - 2sX(s)$$

$$H(s) = \frac{6 - 2s}{2s^2 + 3s + 5}$$

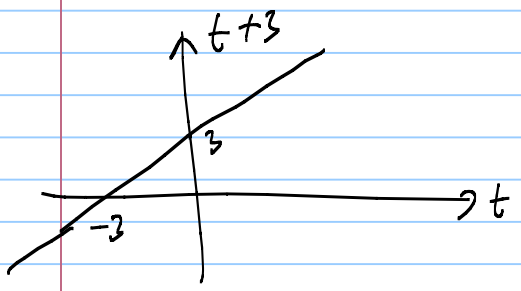
②

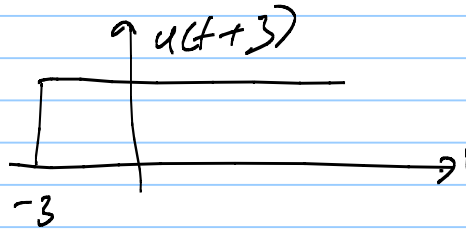
$$2(j\omega)^2 Y(j\omega) + 3(j\omega)Y(j\omega) + 5Y(j\omega) = 6X(j\omega) - 2(j\omega)X(j\omega)$$

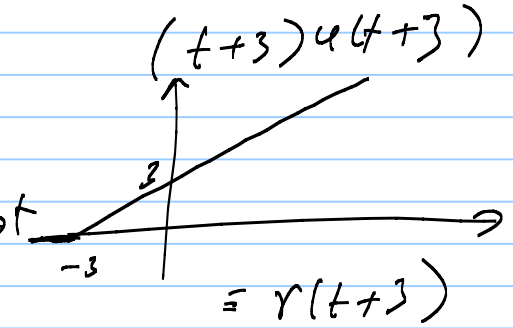
$$H(j\omega) = \frac{6 - 2j\omega}{2(j\omega)^2 + 3j\omega + 5} = \frac{6 - j2\omega}{(5 - 2\omega^2) + j3\omega}$$



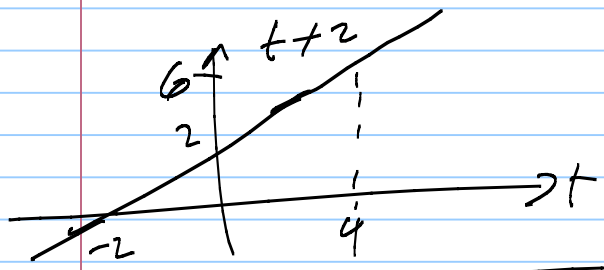
$x(t) = (t+3)\,u(t+3) - 2r(t) + r(t-2) + (t+2)u(t-4)$

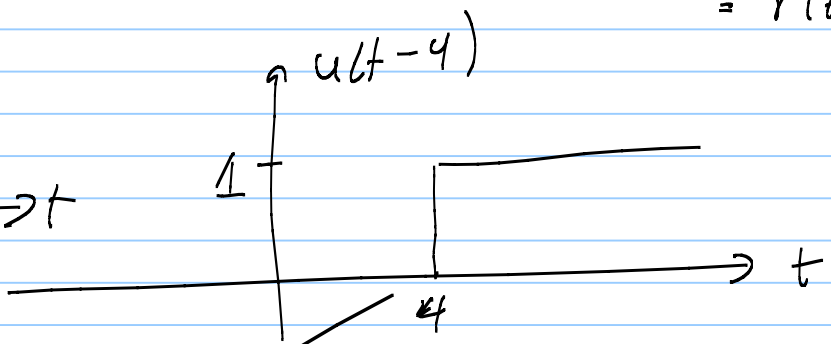


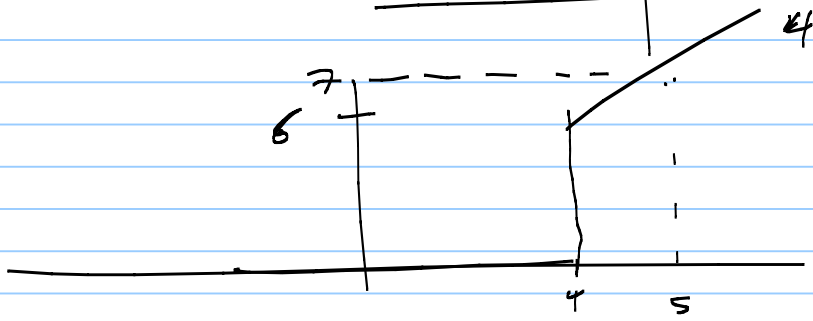




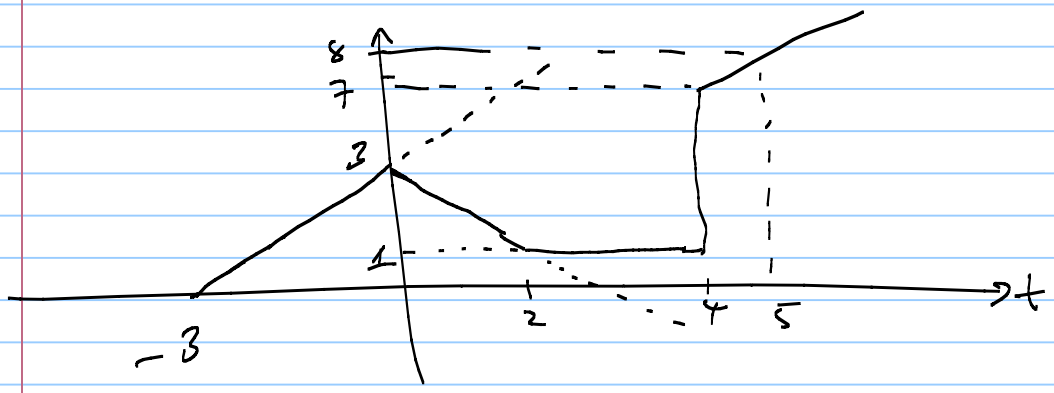
$= r(t+3)$



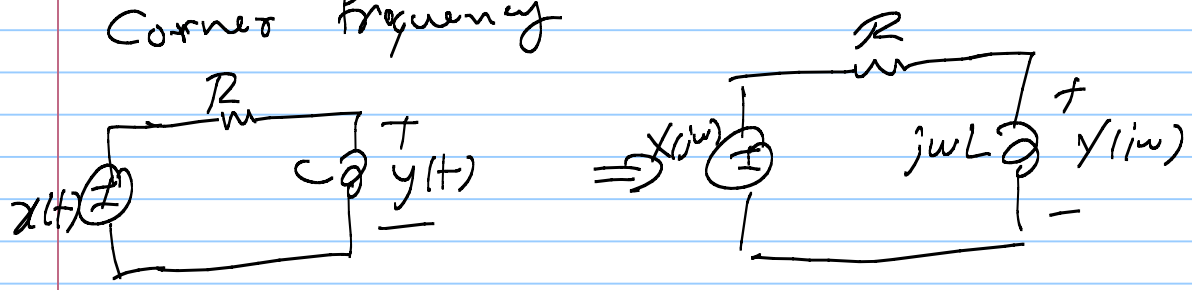




$x(t) = (t+3)\,u(t+3) - 2r(t) + r(t-2) + (t+2)u(t-4)$



Corner frequency



$$Y(j\omega) = X(j\omega)\left(\frac{j\omega L}{R + j\omega L}\right)$$

$$H(j\omega) = \frac{j\omega L}{R + j\omega L} = \frac{j\omega}{R/L + j\omega}, \quad \omega_c = R/L$$

